Present Worth Analysis

Present Worth Analysis of Equal-Life Alternatives

The PW comparison of alternatives with equal lives is straightforward. The present worth *P* is renamed PW of the alternative. The present worth method is quite popular in industry because all future costs and revenues are transformed to **equivalent monetary units NOW;** that is, all future cash flows are converted (discounted) to present amounts (e.g., dollars) at a specific rate of return, which is the MARR. This makes it very simple to determine which alternative has the best economic advantage. The required conditions and evaluation procedure are as follows:

If the alternatives have the same capacities for the same time period (life), the **equal-service requirement** is met. Calculate the PW value at the stated MARR for each alternative.

For **mutually exclusive (ME)** alternatives, whether they are revenue or cost alternatives, the following guidelines are applied to justify a single project or to select one from several alternatives.

One alternative: If PW ≥ 0, the requested MARR is met or exceeded and the alternative is economically justified.

Two or more alternatives: Select the alternative with the PW that is **numerically largest,** that is, less negative or more positive. This indicates a lower PW of cost for cost alternatives or a larger PW of net cash flows for revenue alternatives.

Note that the guideline to select one alternative with the lowest cost or highest revenue uses the criterion of **numerically largest.** This is not the absolute value of the PW amount, because the sign matters.

Example 1

A university lab is a research contractor to NASA for in-space fuel cell systems that are hydrogen and methanol-based. During lab research, three equal-service machines need to be evaluated economically. Perform the present worth analysis with the costs shown below. The MARR is 10% per year.

	Electric-Powered	Gas-Powered	Solar-Powered
First cost, $	−4500	3500	−6000
Annual operating cost (AOC), $/year	−900	−700	−50
Salvage value *S*, $	200	350	100
Life, years	8	8	8

Solution

These are cost alternatives. The salvage values are considered a "negative" cost, so a + sign precedes them. (If it costs money to dispose of an asset, the estimated disposal cost has a - sign.)

The PW of each machine is calculated at i = 10% for n = 8 years. Use subscripts *E, G,* and *S.*

PW_E = -4500 - 900(P/*A, 10*%, 8) + 200(P/*F, 10*%, 8) = $-9208

PW_G = -3500 - 700(P/*A, 10*%, 8) + 350(P/ *F, 10*%, 8) = $-7071

PW_S = -6000 - 50(P/*A, 10*%, 8) + 100(P/*F, 10*%, 8) = $-6220

The solar-powered machine is selected since the PW of its costs is the lowest; it has the numerically largest PW value.

EXAMPLE 2

Ultrapure water (UPW) is an expensive commodity for the semiconductor industry. With the options of seawater or groundwater sources, it is a good idea to determine if one system is more economical than the other. Use a MARR of 12% per year and the present worth method to select one of the systems.

Source	Seawater (S)	Groundwater (G)
Equipment first cost, $M	−20	−22
AOC, $M per year	−0.5	−0.3
Salvage value, % of first cost	5	10
Cost of UPW, $ per 1000 gallons	4	5

Angular has made some initial estimates for the UPW system.

Life of UPW equipment 10 years

UPW needs 1500 gallons/min

Operating time 16 hours per day for 250 days per year

Solution

An important first calculation is the cost of UPW per year. The general relation and estimated costs for the two options are as follows:

UPW cost relation:

$$\frac{\$}{year} = \left(\frac{cost\ in\ \$}{1000\ gallons}\right)\left(\frac{gallons}{minute}\right)\left(\frac{minutes}{hour}\right)\left(\frac{hours}{day}\right)\left(\frac{days}{years}\right)$$

Seawater: (4/1000) (1500) (60) (16) (250) = \$1.44 M per year

Groundwater: (5/1000) (1500) (60) (16) (250) = \$1.80 M per year

Calculate the PW at i = 12% per year and select the option with the lower cost (larger PW value). In \$1 million units:

PW relation:

PW = first cost - PW of AOC - PW of UPW + PW of salvage value

PW_S = -20 - 0.5(*P*/*A*, 12%, 10) - 1.44(*P*/*A*, 12%, 10) + 0.05(20) (*P*/*F*, 12%, 10)

= -20 - 0.5(5.6502) - 1.44(5.6502) + 1(0.3220)

= \$-30.64

PW_G = -22 - 0.3(*P*/*A*, 12%, 10) - 1.80(*P*/*A*, 12%, 10) + 0.10(22) (*P*/*F*, 12%, 10)

= -22 - 0.3(5.6502) - 1.80(5.6502) + 2.2(0.3220)

= \$-33.16

Based on this present worth analysis, the seawater option is cheaper by \$2.52 M.

Present Worth Analysis of Different-Life Alternatives

When the present worth method is used to compare mutually exclusive alternatives that have different lives, the equal-service requirement must be met. The procedure of previous Section is followed, with one exception:

The PW of the alternatives must be compared over the **same number of years** and must end at the same time to satisfy the equal-service requirement

The equal-service requirement is satisfied by using either of two approaches:

LCM: Compare the PW of alternatives over a period of time equal to the **least common multiple (LCM)** of their estimated lives.

Study period: Compare the PW of alternatives using a **specified study period of *n* years.**

This approach does not necessarily consider the useful life of an alternative. The study period is also called the *planning horizon*.

For either approach, calculate the PW at the MARR and use the same selection guideline as that for equal-life alternatives. The LCM approach makes the cash flow estimates extend to the same period, as required. For example, lives of 3 and 4 years are compared over a 12-year period.

The first cost of an alternative is reinvested at the beginning of each life cycle, and the estimated salvage value is accounted for at the end of each life cycle when calculating the PW values over the LCM period. Additionally, the LCM approach requires that some assumptions be made about subsequent life cycles.

The assumptions when using the LCM approach are that

1. The service provided will be needed over the entire LCM years or more.

2. The selected alternative can be repeated over each life cycle of the LCM in exactly the same manner.

3. Cash flow estimates are the same for each life cycle.

EXAMPLE 3

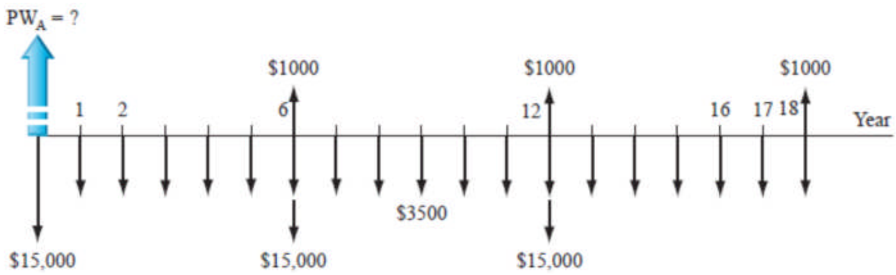
National Homebuilders, Inc., plans to purchase new cut-and-finish equipment. Two manufacturers offered the estimates below.

	Vendor A	Vendor B
First cost, $	−15,000	−18,000
Annual M&O cost, $ per year	−3,500	−3,100
Salvage value, $	1,000	2,000
Life, years	6	9

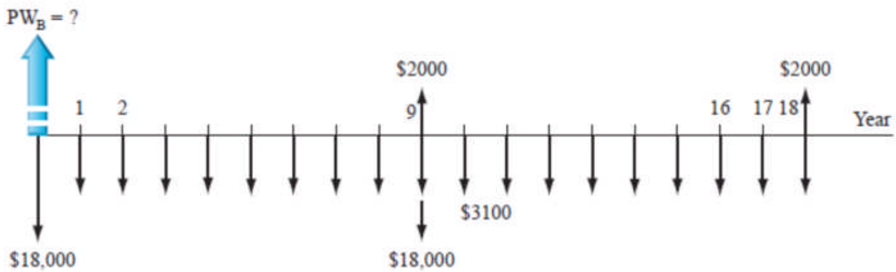
Determine which vendor should be selected on the basis of a present worth comparison, if the MARR is 15% per year.

Solution

Since the equipment has different lives, compare them over the LCM of 18 years. For life cycles after the first, the first cost is repeated in year 0 of each new cycle, which is the last year of the previous cycle. These are years 6 and 12 for vendor A and year 9 for B. The cash flow diagram shown. Calculate PW at 15% over 18 years.



Vendor A



Vendor B

PW_A = -15,000 - 15,000(P/F, 15%, 6) + 1000(P/F, 15%, 6)

- 15,000(P/F, 15%, 12) - 1000(P/F, 15%, 12) + 1000(P/F, 15%, 18)

-3,500(P/A, 15%, 18)

= $ - 45,036

PW_B = -18,000 - 18,000(P/F, 15%, 9) + 2000(P/F, 15%, 9)

+ 2000(P/F, 15%, 18) - 3100(P/A, 15%, 18)

= $-41,384